

Mathematics Standards Guide



**OFFICE OF
CATHOLIC SCHOOLS**



Revised 2015

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REVISION COMMITTEE FOR MATHEMATICS STANDARDS

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The curriculum in the Catholic School should reflect the four interrelated purposes of catechesis: "to proclaim the mysteries of the faith, to foster community, to encourage worship and prayer, and to motivate service to others.

(NCD, #215) Indeed this IS the mission of the Catholic Schools in the Diocese of Owensboro and the Diocese of Lexington.

STANDARDS REVISION PLAN

The Standards Revision Committee for Schools of the Diocese of Owensboro, Kentucky was established in April 1994 to provide direction in developing the curricula for all subject areas. In order to provide direction, there was also a need for a long term, comprehensive standards revision plan. A time line was established for assessing and developing standards that would be consistent with the teachings of the Catholic Church, would address educational reform, and would include the requirements for the Kentucky Non-Public School Certification Process.

Subject Area	Standards Review and Revision	Approval by: Diocesan Standards Committee, PACESS, Committee for Total Catholic Education, Most Rev. William F. Medley	Textbook Selection	Purchase Textbooks
Group I - Math	2014 - 2015 <i>Complete by Spring, 2015</i>	Spring, 2015	<i>Complete by Spring, 2016</i>	Summer, 2016
Group II - English/Language Arts	2015 - 2016 <i>Complete by Spring, 2016</i>	Spring, 2016	<i>Complete by Spring, 2017</i>	Summer, 2017
Group III - Science	2016 - 2017 <i>Complete by Spring, 2017</i>	Spring, 2017	<i>Complete by Spring, 2018</i>	Summer, 2018
Group IV - Social Studies	2017 - 2018 <i>Complete by Spring, 2018</i>	Spring, 2018	<i>Complete by Spring, 2019</i>	Summer, 2019
Group V - Arts & Humanities Foreign Language	2018 - 2019 <i>Complete by Spring, 2019</i>	Spring, 2019	<i>Complete by Spring, 2020</i>	Summer, 2020
Group VI – Religion Vocational Studies and Practical Living	2019 - 2020 <i>Complete by Spring, 2020</i>	Spring, 2020	<i>Complete by Spring, 2021</i>	Summer, 2021

Standards Revision Committee efforts will focus on student learning. It is recommended to teachers that planning, written curriculum guides, textbooks, assessments, etc. be seen as means for student learning. All processes should be developed to ensure continuous improvement of the curriculum.

The Diocesan Standards Revision Committee consensually agreed that the standards for each subject area would be developed and completed according to the standards revision plan. Textbooks and/or materials would be chosen to support the Standards Guide.

It was also agreed that a Standards Revision Committee would be established for each subject. Each committee needs to explore the present status, refer to all available resources, and develop content standards for Grades K-12.

*High Schools Religion Texts can be purchased as they become available and approved by the United States Catholic Conference of Bishops (USCCB).

National Standards and Benchmarks for Effective Catholic Elementary and Secondary Schools

“A school’s Catholic Identity should not be confined to the religion curriculum and campus ministry activities. Every subject taught should be connected in some way to the school’s Catholic Identity. The effective way of aligning academics with Catholic Identity is by integrating the seven principals of Catholic social teaching into the overall school curriculum.”

(Momentum, Sept/Oct 2008)

Standard 2:

An excellent Catholic School adhering to mission provides a rigorous academic program for religious studies and catechesis in the Catholic faith, set within a total academic curriculum that integrates faith, culture and life.

- 2.4** The school’s Catholic Identity requires excellence in academic and intellectual formation in *all* subjects including religious education.
- 2.5** Faculty uses the lenses of Scripture and the Catholic intellectual tradition in *all* subjects to help students think critically and ethically about the world around them.
- 2.7** The theory and practice of the church’ social teachings are essential elements of the curriculum.

Standard 7:

An excellent Catholic school has a clearly articulated rigorous curriculum aligned with relevant standards, 21st century skills and Gospel values, implemented through effective instruction.

- 7.1** The curriculum adheres to appropriate, delineated standards, and is vertically aligned to ensure that every student successfully completes a rigorous and coherent sequence of academic courses based on the standards and *rooted in Catholic values*.
- 7.2** Standards are adopted across the curriculum, and include integration of the religious, spiritual, moral and ethical dimensions of learning in *all* subjects.

MATH PHILOSOPHY

Mathematics plays an integral role in Catholic schools since it reflects the order and unity in God's universe. Mathematics contributes to the formation of Christians who can respond wisely and effectively to a changing world. Contemporary society demands mathematical knowledge which requires students to develop their ability to reason and think logically and to discover creative ways of problem solving. Because of its nature, mathematics can contribute to the development of the whole person by enriching one's life and providing one with a practical tool for daily living.

Toward these ends, students should:

- learn to value mathematics
- learn to reason mathematically
- learn to communicate mathematically
- learn to use technology to investigate and solve problems
- become confident of their mathematical abilities
- become creative mathematical problem solvers

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

PRINCIPLES FOR SCHOOL MATHEMATICS

The seven (7) principles describe crucial issues that are deeply intertwined with school mathematics programs. Each one can influence the development of curriculum frameworks, the selection of curriculum materials, the planning of instructional units or lessons, the design of assessments, the assignments of teachers and students to classes, instructional decisions in the classroom, and the establishment of supportive professional development programs for teachers.

The six principles for school mathematics address overarching themes:

- ***Equity.***
Excellence in mathematics education requires equity – high expectations and strong support for all students.
- ***Curriculum.***
A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics and well-articulated across the grades.
- ***Teaching.***
Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- ***Learning.***
Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- ***Assessment.***
Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- ***Technology.***
Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.
- ***Professionalism.***
In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics.

National Council of Teachers of Mathematics (2014) Principles to Actions. Reston, VA: NCTM, Inc.

DIOCESE OF LEXINGTON ACADEMIC MATH STANDARDS

KINDERGARTEN

DOMAINS:

Counting and Cardinality

- K.CC.1* Count to 100 by ones and by tens.
- K.CC.2* Count forward from a given number within the known sequence.
- K.CC.3* Represent a number of objects with a numeral from 0-20 verbally and in writing.
- K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
- K.CC.4a* Count objects by using the number name in the standard order understanding that the number name relates to the quantity counted no matter how the objects are arranged.
- K.CC.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- K.CC.4c* Understand that each successive number name refers to a quantity that is one larger than the number name that came before it.
- K.CC.6* Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group by using matching and counting strategies.
- K.CC.7* Compare written numbers up to 10 and arrange in numerical order.
- K.CC.8 Identify simple fractions ($\frac{1}{2}$, whole) using pictures.
- K.CC.8a Recognize parts of a fraction as whole and half.

Operations and Algebraic Thinking

- K.OA.1* Explain verbally and model solving routine and non-routine addition and subtraction problems by using manipulatives, drawing diagrams, and acting them out.
- K.OA.2* Solve addition and subtraction word problems within 10 by using manipulatives or drawings to represent the problem.
- K.OA.3* Decompose numbers less than or equal to 10 into pairs in more than one way.
- Example: by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $6 = 3 + 3$, $6 = 2 + 4$, $6 = 1 + 5$).
- K.OA.4* For any number from 1 to 9, find the number that makes 10 when added to the given number.
- Example: Use objects or drawings, and record the answer with a drawing or equation.
- K.OA.5* Fluently add and subtract within 5.
- K.O.A.5a Fluently add doubles for any number 1-5.
- K.OA.6 Extend, describe, and create patterns using pictures, objects, sounds, movement.

Number and Operations in Base Ten

K.NBT.1* Compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $19 = 10 + 9$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

K.MD.1* Compare and order objects indirectly or directly using measurable attributes such as length, height and weight.

K.MD.2* Directly compare two objects with a measurable attribute in common, to see which object has more of/less of the attribute, and describe the difference.

- Example: Compare the heights of two children and describe one child as taller/shorter.

K.MD.3* Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10).

K.MD.4 Demonstrate an understanding of the concept of time, using identifiers such as morning, afternoon, day, week, month, year, before/after, and shorter/longer. K.MD.5 Identify the name and value of a penny, nickel, dime, and quarter.

K.MD.6 Use appropriate tools to measure.

K.MD.7 Organize, represent and interpret data using pictures, graphs, tally charts, and bar graphs.

K.MD.8 Use graphs to answer questions.

Geometry

K.G.1* Describe objects in the physical world using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

K.G.2* Correctly name shapes regardless of their orientations or overall size.

K.G.3* Identify shapes as two-dimensional or three-dimensional.

- Example: lying in a plane, ‘flat’ - squares, triangles, circles, rectangles, ovals, hexagons
- Example: ‘solid’ - spheres, cubes, cylinders

K.G.4* Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts, and other attributes.

- Example: number of sides and vertices/corners.
- Example: having sides of equal length.

K.G.5* Model shapes in the world by building shapes from components and drawing shapes.

- Example: sticks and clay balls

K.G.6* Compose simple shapes to form larger shapes.

- Example: Join two triangles with full sides touching to create a rectangle.

K.G.7 Recognize equal parts of a whole

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 1

Operations and Algebraic Thinking

- 1.OA.1* Use addition and subtraction within 20 to solve word problems involving situations of *adding to*, *taking from*, *putting together*, *taking apart*, and *comparing*, with unknowns in all positions.
- Example: Use objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- 1.OA.2* Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.
- 1.OA.3* Apply properties of operations as strategies to add and subtract.
- Example: Commutative property of addition: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known.
 - Example: Associative property of addition: To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$.
- 1.OA.4* Understand subtraction as an unknown-addend problem.
- Example: Subtract $10 - 8$ by finding the number that makes 10 when added to 8.
- 1.OA.5*
- Relate counting to addition and subtraction.
 - Example: Count on 2 to add 2.
- 1.OA.6* Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as:
- Making ten ($8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$)
 - Using the relationship between addition and subtraction (knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$).
 - Creating equivalent but easier or known sums (adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).
- 1.OA.7* Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.
- Example: Which of the following equations are true and which are false?
 $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$
- 1.OA.8* Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.
- Example: Determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = ? - 3$, $6 + 6 = ?$
- 1.OA.9 Create and extend number patterns using addition and subtraction.
- 1, 3, 5, ..., (tell the next numbers in the pattern and why, also 10, 7, 4, ...).

Number and Operations in Base Ten

- 1.NBT.1* Count, read, order, estimate, and write numerals up to and including 120, and represent a number of objects within this range with a written numeral.
- 1.NBT.2* Understand that the two digits of a two-digit number represent amounts of tens and ones.
- 1.NBT.2a 10 can be thought of as a bundle of ten ones – called a “ten.”
- 1.NBT.2b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- 1.NBT.2c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
- 1.NBT.3* Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparison with the symbols $>$, $=$, and $<$.
- 1.NBT.4* Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10 using concrete models or drawings and strategies based on place value.
- 1.NBT.5* Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count.
- 1.NBT.6* Subtract multiples of ten in the range 10-90 from multiples of 10 in the range 10-90, using concrete models or drawings and strategies based on place value.

Measurement and Data

- 1.MD.1* Order three objects by length.
- 1.MD.1* Compare the lengths of two objects indirectly by using a third object.
- 1.MD.2* Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
- 1.MD.3* Tell and write time in hours and half-hours using analog and digital clocks
- 1.MD.4* Organize, represent, and interpret data with up to three categories using charts, tables, pictographs, and bar graphs
- 1.MD.5 Ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
- 1.MD.6 Fluently identify the value of penny, nickel, dime, quarter, half dollar and dollar.

Geometry

- 1.G.1* Distinguish between defining attributes or characteristics (e.g., triangles are closed and three-sided) and non-defining attributes or characteristics (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes or characteristics.
- 1.G.2* Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
- 1.G.3* Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 2

Operations and Algebraic Thinking

- 2.OA.1* Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.
- Example: Use drawings and equations with a symbol for the unknown number to represent the problem.
- 2.OA.2* Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.
- 2.OA.3* Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or count them by 2's; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4* Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
- 2.OA.5 Extend and create patterns with more than two attributes.

Number and Operations in Base Ten

- 2.NBT.1* Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
- 2.NBT.1a* Understand 100 can be thought of as a bundle of ten tens – called a “hundred.”
- 2.NBT.1b* Understand the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- 2.NBT.2* Count within 1000; skip-count by 5's, 10's and 100's
- 2.NBT.3* Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- 2.NBT.4* Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.
- 2.NBT.5* Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationships between addition and subtraction.
- 2.NBT.6* Add up to four two-digit numbers using strategies based on place value and properties of operations.

- 2.NBT.7* Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
- 2.NBT.8* Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
- 2.NBT.9* Explain why addition and subtraction strategies work, using place value and the properties of operations.
- 2.NBT.10 Model using arrays basic multiplication concepts for 2, 5, and 10
- 2.NBT.11 Draw and compare fractions using models and pictures.

Measurement and Data

- 2.MD.1* Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2.MD.2* Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- 2.MD.3* Estimate lengths using units of inches, feet, centimeters, and meters
- 2.MD.4* Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
- 2.MD.5* Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units; e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
- 2.MD.6* Represent whole numbers as lengths, from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
- 2.MD.7* Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
- 2.MD.8* Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and cents symbols appropriately.
 - Example: If you have 2 dimes and 3 pennies, how many cents do you have?
- 2.MD.9* Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
- 2.MD.10* Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
- 2.MD.11 Interpret data to predict probability.

2.MD.12 Analyze and use the calendar.

Geometry

- 2.G.1* Recognize and draw shapes having specified attributes, such as a given number of vertices, angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
- 2.G.2* Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
- 2.G.3* Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
- 2.G.4 Identify patterns, symmetry, and congruency.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 3

Operations and Algebraic Thinking

- 3.OA.1* Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.
- Example: Describe a context in which a total number of objects can be expressed as 5×7 .
- 3.OA.2* Interpret whole-number quotients of whole numbers.
- Example: Interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.
 - Example: Describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.
- 3.OA.3 * Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4 * Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- Example: Determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$
- 3.OA.5 * Apply properties of operations as strategies to multiply and divide.
- Example: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.)
 - $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)
 - Example: Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.6 * Understand division as an unknown-factor problem.
- Example: Find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.
- 3.OA.7 * Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products and quotients of basic facts from 0-12.
- 3.OA.8 * Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- 3.OA.9* Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Number and Operations in Base Ten

- 3.NBT.1* Use place value understanding to round whole numbers to the nearest 10 or 100.
- 3.NBT.2* Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 3.NBT.3* Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Numbers and Operations-Fractions

- 3.NF.1* Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
- 3.NF.2* Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- 3.NF.2a* Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
- 3.NF.2b* Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
- 3.NF.3* Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- 3.NF.3a* Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- 3.NF.3b* Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- 3.NF.3c* Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
- Example: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.
- 3.NF.3d* Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, $<$, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data

- 3.MD.1* Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- 3.MD.1a Calculate elapsed time in minutes and hours.
- 3.MD.2* Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
- 3.MD.2a Measure and estimate liquid volumes using units of cups, pints, quarts, and gallons.
- 3.MD.3* Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.
- Example: Draw a bar graph in which each square in the bar graph might represent 5 pets.
- 3.MD.4* Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.
- 3.MD.4a Measure lengths using units of inches, feet, centimeters, and meters.
- 3.MD.5* Recognize area as an attribute of plane figures and understand concepts of area measurement.
- 3.MD.5a* A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
- 3.MD.5b* A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
- 3.MD.6* Measure areas by counting unit squares (square cm., square m., square in., square ft., and improvised units).
- 3.MD.7* Relate area to the operations of multiplication and addition.
- 3.MD.7a* Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- 3.MD.7b* Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- 3.MD.7c* Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a + b$ and $a + c$. Use area models to represent the distributive property in mathematical reasoning.
- 3.MD.7d* Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

- 3.MD.8* Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
- 3.MD.9* Read and interpret temperature using Celsius and Fahrenheit scales.

Geometry

- 3.G.1* Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- 3.G.2* Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
- Example: Partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 4

Operations and Algebraic Thinking

- 4.OA.1 * Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
- 4.OA.2 * Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
- 4.OA.3 * Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- 4.OA.3a Explore order of operations.
- 4.OA.4* Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number.
- 4.OA.4a* Determine whether a given whole number in the range 1-100 is prime or composite.
- 4.OA.5 * Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
- Example: Given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
- 4.OA.6 Show remainders as fractions and decimals.
- 4.OA.7 Recognize and determine greatest common factor (GCF) and least common multiple (LCM) and interpret remainders in problem solving.

Number and Operations in Base Ten

- 4.NBT.1* Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
- Example: Recognize that $700 \div 70 = 10$ by applying concepts of place value and division.
- 4.NBT.2* Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, $<$ symbols to record the results of comparisons.
- 4.NBT.3* Use place value understanding to round multi-digit whole numbers to any place.

- 4.NBT.4* Fluently add and subtract multi-digit whole numbers using the standard algorithm.
- 4.NBT.5* Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations, illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- 4.NBT.6* Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division, illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations-Fractions

- 4.NF.1* Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- 4.NF.2* Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, $<$, and justify the conclusions, e.g., by using a visual fraction model.
- 4.NF.3* Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
- 4.NF.3a* Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- 4.NF.3b* Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.
- 4.NF.3c* Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- 4.NF.3d* Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
- 4.NF.4* Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
- 4.NF.4a* Understand a fraction a/b as a multiple of $1/b$.
- Example: Use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

- 4.NF.4b* Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.
- Example: Use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)
- 4.NF.4c* Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- Example: If each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
- 4.NF.5* Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.
- Example: Express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$
- 4.NF.6* Use decimal notation for fractions with denominators 10 or 100.
- Example: Rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
- 4.NF.7* Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.
- 4.NF.8 Apply greatest common factor (GCF) to express sums and differences in simplest form.

Measurement and Data

- 4.MD.1* Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.
Record measurement equivalents in a two-column table.
- Example: Know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
- 4.MD.2* Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
- 4.MD.3* Apply the area and perimeter formulas for rectangles in real world and mathematical problems.
- Example: Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

- 4.MD.4* Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
- Example: From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.
- 4.MD.5* Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
- 4.MD.5a* An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.
- 4.MD.5b* An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
- 4.MD.6* Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
- 4.MD.7* Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
- 4.MD.8 Define and find the mean, median, and mode of a set of data.

Geometry

- 4.G.1* Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- 4.G.2* Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- 4.G.3* Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 5

Operations and Algebraic Thinking

- 5.OA.1 * Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- 5.OA.2* Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
- Example: Express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.
- 5.OA.3 * Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
- Example: Given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Number and Operations in Base Ten

- 5.NBT.1* Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.
- 5.NBT.2* Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- 5.NBT.3* Read, write, and compare decimals to thousandths.
- 5.NBT.3a* Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
- 5.NBT.3b* Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.
- 5.NBT.4* Use place value understanding to round decimals to any place.
- 5.NBT.5* Fluently multiply multi-digit whole numbers using the standard algorithm.
- 5.NBT.6* Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- 5.NBT.7* Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations-Fractions

- 5.NF.1* Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
- Example: $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)
- 5.NF.2* Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
- Example: Recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.
- 5.NF.3* Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- Example: Interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among people each person has a share of size $3/4$. If 9 people want to share a 50pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
- 5.NF.4* Apply and extend previous understanding so of multiplication to multiply a fraction or whole number by a fraction.
- 5.NF.4a* Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.
- Example: Use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
- 5.NF.4b* Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- 5.NF.5* Interpret multiplication as scaling (resizing).
- 5.NF.5a* Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

- 5.NF.5b* Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
- 5.NF.6* Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- 5.NF.7* Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
- 5.NF.7a* Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
- Example: Create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
- 5.NF.7b* Interpret division of a whole number by a unit fraction, and compute such quotients.
- Example: Create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
- 5.NF.7c* Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models an equations to represent the problem.
- Example: How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $1/3$ -cup serving share in 2 cups of raisins?

Measurement and Data

- 5.MD.1* Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
- 5.MD.2* Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Use operations on fractions for this grade to solve problems involving information presented in line plots.
- Example: Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
- 5.MD.3* Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- 5.MD.3a* A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

- 5.MD.3b* A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- 5.MD.4* Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.5* Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 5.MD.5a* Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- 5.MD.5b* Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- 5.MD.5c* Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
- 5.MD.6 Collect, organize and interpret data.
- 5.MD.7 Calculate and apply range, median, mode, and mean with whole numbers.

Geometry

- 5.G.1* Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x -axis and x -coordinate, y -axis and y -coordinate).
- 5.G.2* Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
- 5.G.2a Graph points, parallel and perpendicular lines, rays and angles in the first quadrant of the coordinate plane.
- 5.G.3* Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
- Example: All rectangles have four right angles and squares are rectangles, so all squares have four right angles.
- 5.G.4* Classify two-dimensional figures in a hierarchy based on properties.
- 5.G.5 Recognize and apply slides, flips and turns.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 6

Ratios and Proportional Relationships

- 6.RP.1* Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- Example: “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
- 6.RP.2* Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
- Example: “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”
- 6.RP.3* Use ratio and rate reasoning to resolve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- Example: Reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.3a* Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- 6.RP.3b* Solve unit rate problems including those involving unit pricing and constant speed.
- Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- 6.RP.3c* Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
- Example: 30% of a quantity means 30/100 times the quantity.
- 6.RP.3d* Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

- 6.NS.1* Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
- Example: Create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How wide is a rectangular strip of land with length $\frac{3}{4}$ mile and area $\frac{1}{2}$ square mile?

- 6.NS.2* Fluently divide multi-digit numbers using the standard algorithm.
- 6.NS.3* Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- 6.NS.4* Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.
- 6.NS.4a* Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- Example: Express $36 + 8$ as $4(9 + 2)$.
- 6.NS.5* Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g. temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- Example: Temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electrical charge.
- 6.NS.6* Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- 6.NS.6a* Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- 6.NS.6b* Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- 6.NS.6c* Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- 6.NS.7* Understand ordering and absolute value of rational numbers.
- 6.NS.7a* Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- Example: Interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.
- 6.NS.7b* Write, interpret, and explain statements of order for rational numbers in realworld contexts.
- Example: Write $-3\text{ }^{\circ}\text{C} > -7\text{ }^{\circ}\text{C}$ to express the fact that $-3\text{ }^{\circ}\text{C}$ is warmer than $-7\text{ }^{\circ}\text{C}$.
- 6.NS.7c* Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- Example: For an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

- 6.NS.7d* Distinguish comparisons of absolute value from statements about order.
- Example: Recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.
- 6.NS.8* Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- 6.NS.9 Convert fractions to decimals and decimals to fractions.

Expressions and Equations

- 6.EE.1* Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.2* Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.2a* Write expressions that record operations with numbers and with letters standing for numbers.
- Example: Express the calculation “Subtract y from 5” as “ $5 - y$.”
- 6.EE.2b* Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
- Example: Describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.
- 6.EE.2c* Evaluate expressions as specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
- Example: Use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.EE.3* Apply the properties of operations to generate equivalent expressions. □ Example: Apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- 6.EE.4* Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).
- Example: The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- 6.EE.5* Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.6* Use variables to represent numbers and write expressions when solving a real world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- 6.EE.7* Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
- 6.EE.8* Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- 6.EE.9* Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
- Example: In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Geometry

- 6.G.1* Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.2* Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l \cdot w \cdot h$ and $V = b \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving realworld and mathematical problems.
- 6.G.3* Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.4* Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.5 Identify, describe, classify, name and draw pairs of angles (adjacent, vertical, complementary, supplementary, and alternate interior and alternate exterior angles).
- 6.G.6 Apply the properties of dilations, rotations, reflections, and translations to two-dimensional figures.

Statistics and Probability

- 6.SP.1* Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
- Example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
- 6.SP.2* Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6.SP.3* Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- 6.SP.4* Display numerical data in plots on a number line, including dot plots, histograms, and box plots (e.g., box-and-whisker plot).
- 6.SP.5* Summarize numerical data sets in relation to their context.
- 6.SP.5a* Report the number of observations.
- 6.SP.5b* Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
- 6.SP.5c* Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
- 6.SP.5d* Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- 6.SP.5e Interpret the meaning of fractional and decimal values as related to mean.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 7

Ratios and Proportional Relationships

- 7.RP.1* Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- Example: If a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour. The solution would be 2 miles per hour.
- 7.RP.2* Recognize and represent proportional relationships between quantities.
- 7.RP.2a* Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.2b* Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.2c* Represent proportional relationships by equations.
- Example: If total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.
- 7.RP.2d* Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- 7.RP.3* Use proportional relationships to solve multistep ratio and percent problems.
- Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- 7.RP.4 Convert between decimal, fraction, and percent formats.
- 7.RP.5 Compare and order percents (including those less than one and greater than 100).
- 7.RP.6 Solve ratio equations using cross-multiplication.
- 7.RP.7 Solve word problems involving ratios and proportions, including the percent proportion (e.g., 16 is what percent of 90).
- 7.RP.8 Apply ratios and solve problems involving scale, models, and unit rates.

The Number System

- 7.NS.1* Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- 7.NS.1a* Describe situations in which opposite quantities combine to make 0.
- Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.
- 7.NS.1b* Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.

- 7.NS.1c* Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7.NS.1d* Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7.NS.1e* Apply properties of operations as strategies to add and subtract rational numbers.
- 7.NS.2* Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- 7.NS.2a* Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- 7.NS.2b* Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- 7.NS.2c* Apply properties of operations as strategies to multiply and divide rational numbers.
- 7.NS.2d* Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.NS.3* Solve real-world and mathematical problems involving the four operations with rational numbers.
- 7.NS.4 Identify, order and compare integers and rational numbers.

Expressions and Equations

- 7.EE.1* Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.2* Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.
- Example: $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”
- 7.EE.3* Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using appropriate tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- Example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9 \frac{3}{4}$ inches long in the center of a door that is $27 \frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check for the exact computation.

- 7.EE.4* Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.4a* Solve word problems leading to equations of the form $px + pq = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
- 7.EE.4b* Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
- Example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.
- 7.EE.5 Identify and combine like terms (e.g., $2x + 3x = 5x$).
- 7.EE.6 Solve and check two-step equations (e.g., $2x + 3 = 5$) using rational numbers and the distributive property [$2(x + 3) = 8$].
- 7.EE.7 Evaluate solutions for reasonableness, accuracy, and completeness.

Geometry

- 7.G.1* Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.2* Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.G.3* Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- 7.G.4* Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.5* Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- 7.G.6* Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.G.7 Prove the similarity of plane figures by identifying congruent angles and proportional sides.
- 7.G.8 Verify the properties of dilations, rotations, reflections, and translations.

7.G.8a Use the properties of dilations, rotations, reflections, and translations to compare two-dimensional figures.

Statistics and Probability

7.SP.1* Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.2* Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

- Example: Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

7.SP.3* Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

- Example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

7.SP.4* Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

- Example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth grade science book.

7.SP.5* Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6* Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Predict the approximate relative frequency given the probability.

- Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7* Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- 7.SP.7a* Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
- 7.SP.7b* Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
- Example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- 7.SP.8* Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- 7.SP.8a* Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- 7.SP.8b* Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- 7.SP.8c* Design and use a simulation to generate frequencies for compound events.
- Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
- 7.SP.9 Differentiate between theoretical and experimental probability.
- 7.SP.10 Predict, infer and create line plots, histograms, stem and leaf, bar graphs, box plots.
- 7.SP.11 Use random sampling to draw inferences about a population.
- 7.SP.12 Draw informal comparative inferences about two populations.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

GRADE 8

The Number System

- 8.NS.1* Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- 8.NS.2* Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
- Example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations

- 8.EE.1* Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- Example: $3^2 \times 3^{-5} = 1/3^3 = 1/27$.
- 8.EE.2* Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- 8.EE.3* Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- Example: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.4* Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notations are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).
- 8.EE.4a* Interpret scientific notation that has been generated by technology.
- 8.EE.5* Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- 8.EE.6* Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- 8.EE.7* Solve linear equations in one variable.

- 8.EE.7a* Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- 8.EE.7b* Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 8.EE8* Analyze and solve pairs of simultaneous linear equations.
- 8.EE8a* Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- 8.EE8b* Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- Example: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
- 8.EE8c* Solve real-world and mathematical problems leading to two linear equations in two variables.
- Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Functions

- 8.F.1* Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- 8.F.2* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
- 8.F.3* Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- Example: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.
- 8.F.4* Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- 8.F.5* Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

- 8.G.1* Verify experimentally the properties of rotations, reflections, and translations:
- 8.G.1a* Verify lines are taken to lines, and line segments to line segments of the same length.
- 8.G.1b* Verify angles are taken to angles of the same measure.
- 8.G.1c* Verify parallel lines are taken to parallel lines.
- 8.G.2* Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.3* Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.4* Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8.G.5* Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
- Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
- 8.G.6* Explain a proof of the Pythagorean Theorem and its converse.
- 8.G.7* Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.8* Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- 8.G.9* Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
- 8.G.10 Apply the area formula to complex figures in a real world context using rational numbers.

Statistics and Probability

- 8.SP.1* Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

- 8.SP.2* Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.3* Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
- 8.SP.4* Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
- Example: Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

7th GRADE PRE-ALGEBRA

<p style="text-align: center;">Unit 1: Rational Numbers and Exponents</p>

The Number System

- 7.NS.1* Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- 7.NS.1a* Describe situations in which opposite quantities combine to make 0.
- Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.
- 7.NS.1b* Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative.
- 7.NS.1c* Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7.NS.1d* Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7.NS.1e* Apply properties of operations as strategies to add and subtract rational numbers.
- 7.NS.2* Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- 7.NS.2a* Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- 7.NS.2b* Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- 7.NS.2c* Apply properties of operations as strategies to multiply and divide rational numbers.
- 7.NS.2d* Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.NS.3* Solve real-world and mathematical problems involving the four operations with rational numbers.

- 8.NS.1* Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- 8.NS.2* Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
- Example: By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Expressions and Equations

- 8.EE.1* Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- Example: $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.EE.2* Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- 8.EE.3* Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- Example: Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.4* Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).
- 8.EE.4a Interpret scientific notation that has been generated by technology.

Unit 2:
Proportionality and Linear Relationships

Ratios and Proportional Relationships

- 7.RP.1* Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
- Example: If a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $1/2 / 1/4$ miles per hour. The solution would be 2 miles per hour.
- 7.RP.2* Recognize and represent proportional relationships between quantities.

- 7.RP.2a* Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.2b* Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.2c* Represent proportional relationships by equations.
- Example: If total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.
- 7.RP.2d* Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- 7.RP.3* Use proportional relationships to solve multistep ratio and percent problems.
- Examples: Simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Expressions and Equations

- 7.EE.1* Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.2* Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.
- Example: $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”
- 7.EE.3* Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using appropriate tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- Example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check for the exact computation.
- 7.EE.4* Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.4a* Solve word problems leading to equations of the form $px + pq = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

- EE.4b* Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
- Example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.
- EE.5* Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- 8.EE.6* Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- 8.EE.7* Solve linear equations in one variable.
- 8.EE.7a* Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- 8.EE.7b* Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
- 8.EE8* Analyze and solve pairs of simultaneous linear equations.
- 8.EE8a* Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- 8.EE8b* Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
- Example: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
- 8.EE8c* Solve real-world and mathematical problems leading to two linear equations in two variables.
- Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Unit 3: Introduction to Sampling and Inference

Statistics and Probability

- 7.SP.1* Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.2* Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- Example: Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.3* Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
- Example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
- 7.SP.4* Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.
- Example: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth grade science book.
- 7.SP.5* Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.6* Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Predict the approximate relative frequency given the probability.
- Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- 7.SP.7* Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- 7.SP.7a* Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
- Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected
- 7.SP.7b* Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
- Example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- 7.SP.8* Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- 7.SP.8a* Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- 7.SP.8b* Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- 7.SP.8c* Design and use a simulation to generate frequencies for compound events.
- Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
- 7.SP.10 Predict, infer and create line plots, histograms, stem and leaf, bar graphs, box plots.
- 7.SP.11 Use random sampling to draw inferences about a population.
- 7.SP.12 Draw informal comparative inferences about two populations.

Unit 4:

Creating, Comparing, and Analyzing Geometric Figures

Geometry

- 7.G.1* Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.2* Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

- 7.G.3* Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- 7.G.4* Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.5* Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- 7.G.6* Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.G.7* Verify experimentally the properties of rotations, reflections, and translations:
- 8.G.1a* Verify lines are taken to lines, and line segments to line segments of the same length.
- 8.G.1b* Verify angles are taken to angles of the same measure.
- 8.G.1c* Verify parallel lines are taken to parallel lines.
- 8.G.2* Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.3* Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.4* Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8.G.5* Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
 - Example: Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.
- 8.G.6* Explain a proof of the Pythagorean Theorem and its converse.
- 8.G.7* Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.8* Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- 8.G.9* Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

8th GRADE ALGEBRA I

Unit 1:

Relationships between Quantities and Reasoning with Equations

Quantities

- N-Q.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.2* Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Seeing Structure in Expressions

- A-SSE.1* Interpret expressions that represent a quantity in terms of its context.
- A-SSE.1a* Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.1b* Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P .

Creating Equations

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.2* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3* Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.
- A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Example: Rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities

- A-REI.1* Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- A-REI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 2:

Linear and Exponential Functions

The Real Number System

- N-RN.1* Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- Example: we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
- N-RN.2* Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Reasoning with Equations and Inequalities

- A-REI.5* Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.6* Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.10* Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A-REI.11* Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.12* Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

- 8.F.1* Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- 8.F.2* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
- 8.F.3* Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- Example: The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

Interpreting Functions

- F-IF.1* Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.2* Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.3* Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Example: The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
- 8.F.4* Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.5* Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- F-IF.4* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

- F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-IF.7a* Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.7e* Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

- F.BF.1* Write a function that describes a relationship between two quantities.
- F.BF.1a* Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.1b* Combine standard function types using arithmetic operations.
- Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F.BF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula. Use them to model situations, and translate between the two forms.
- F.BF.3* Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Linear and Exponential Models

- F.LE.1* Distinguish between situations that can be modeled with linear functions and with exponential functions.
- F.LE.1a* Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F.LE.1b* Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- F-LE.1c* Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F-LE.5* Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Descriptive Statistics

Interpreting Categorical and Quantitative Data

- S-ID.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).
- S-ID.2* Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- S-ID.3* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Statistics and Probability

- 8.SP.1* Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SP.2* Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.3* Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Example: In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
- 8.SP.4* Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.
- Example: Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Interpreting Categorical and Quantitative Data

- S-ID.5* Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- S-ID.6* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- S-ID.6a* Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
- S-ID.6b* Informally assess the fit of a function by plotting and analyzing residuals.
- S-ID.6c* Fit a linear function for a scatter plot that suggests a linear association.
- S-ID.7* Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- S-ID.8* Compute (using technology) and interpret the correlation coefficient of a linear fit.
- S-ID.9* Distinguish between correlation and causation.

Unit 4: Expressions and Equations
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Seeing Structure in Expressions

- A-SSE.1* Interpret expressions that represent a quantity in terms of its context.
- A-SSE.1a* Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.1b* Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Example: interpret $P(1+r)^n$ as the product of P and a factor not depending on P .
- A-SSE.2* Use the structure of an expression to identify ways to rewrite it.
- Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
- A-SSE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- A-SSE.3a* Factor a quadratic expression to reveal the zeros of the function it defines.
- A-SSE.3b* Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

- A-SSE.3c* Use the properties of exponents to transform expressions for exponential functions.
- Example: The expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Arithmetic with Polynomials and Rational Expressions

- A-APR.1* Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.2* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Example: Rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities

- A-REI.4* Solve quadratic equations in one variable.
- A-REI.4a* Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4b* Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- A-REI.7* Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- Example: Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Note: A-SSE.3a, A-SSE.3b and A-SSE.3c not in Algebra I high school standards

Unit 5: Quadratic Functions and Modeling

The Real Number System

N-RN.3* Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpreting Functions

F-IF.4* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.7a* Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.7b* Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.8* Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.8a* Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

- F-IF.8b* Use the properties of exponents to interpret expressions for exponential functions.
- Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
- F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: Given a graph of one quadratic function and an algebraic expression for another, identify which has the larger maximum.

Building Functions

- F.BF.1* Write a function that describes a relationship between two quantities.
- F.BF.1a* Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.1b* Combine standard function types using arithmetic operations.
- Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F.BF.3* Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- F.BF.4* Find inverse functions.
- F.BF.4a* Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear and Exponential Models

- F-LE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

*From: NCTM, *CCSS, ITBS, *KDE, Archdioceses of *Louisville, *Cincinnati and *Columbus Standards*

HIGH SCHOOL ALGEBRA I

Unit 1:

Relationships between Quantities and Reasoning with Equations

Quantities

- N-Q.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- N-Q.2* Define appropriate quantities for the purpose of descriptive modeling.
- N-Q.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Seeing Structure in Expressions

- A-SSE.1* Interpret expressions that represent a quantity in terms of its context.
- A-SSE.1a* Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.1b* Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Example: interpret $P(1+r)^n$ as the product of P and a factor not depending on P .

Creating Equations

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.2* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3* Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- Example: represent inequalities describing nutritional and cost constraints on combinations of different foods.
- A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Example: rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities

- A-REI.1* Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- A-REI.3* Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

<p style="text-align: center;">Unit 2: Linear and Exponential Relationships</p>

The Real Number System

- N-RN.1* Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- N-RN.2* Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Reasoning with Equations and Inequalities

- A-REI.5* Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.6* Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.10* Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A-REI.11* Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.12* Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions

- F-IF.1* Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- F-IF.2* Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.3* Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- Example: the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
- F-IF.4* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Example: if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-IF.7a* Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.7e* Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

- F.BF.1* Write a function that describes a relationship between two quantities.
- F.BF.1a* Determine an explicit expression, a recursive process, or steps for calculation from a context.

- F.BF.1b* Combine standard function types using arithmetic operations.
- Example: build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F.BF.2* Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F.BF.3* Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Linear and Exponential Models

- F-LE.1* Distinguish between situations that can be modeled with linear functions and with exponential functions.
- F-LE.1a* Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- F-LE.1b* Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- F-LE.1c* Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- F-LE.5* Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3:
Descriptive Statistics

Interpreting Categorical and Quantitative Data

- S-ID.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).
- S-ID.2* Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- S-ID.3* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- S-ID.5* Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- S-ID.6* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
- S-ID.6a* Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
- S-ID.6b* Informally assess the fit of a function by plotting and analyzing residuals.
- S-ID.6c* Fit a linear function for a scatter plot that suggests a linear association.
- S-ID.7* Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- S-ID.8* Compute (using technology) and interpret the correlation coefficient of a linear fit.
- S-ID.9* Distinguish between correlation and causation.

Unit 4:
Expressions and Equations

Seeing Structure in Expressions

- A-SSE.1* Interpret expressions that represent a quantity in terms of its context.
- A-SSE.1a* Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.1b* Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Example: interpret $P(1+r)^n$ as the product of P and a factor not depending on P .

- A-SSE.2* Use the structure of an expression to identify ways to rewrite it.
- Example: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
- A-SSE.3* Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Arithmetic with Polynomials and Rational Expressions

- A-APR.1* Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.2* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Example: rearrange Ohm's law $V = IR$ to highlight resistance R .

Reasoning with Equations and Inequalities

- A-REI.4* Solve quadratic equations in one variable.
- A-REI.4a* Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- A-REI.4b* Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- A-REI.7* Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
- Example: find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Unit 5: Quadratic Functions and Modeling

The Real Number System

N-RN.3* Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpreting Functions

F-IF.4* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- Example: if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF.7a* Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.7b* Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.8* Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF.8a* Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.8b* Use the properties of exponents to interpret expressions for exponential functions.

- Example: Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- Example: given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

- F.BF.1* Write a function that describes a relationship between two quantities.
- F.BF.1a* Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.1b* Combine standard function types using arithmetic operations.
- Example: build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F.BF.3* Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4* Find inverse functions.
- F.BF.4a* Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Geometry

- F-LE-3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

*From: NCTM, *CCSS, *KDE, Archdiocese of *Cincinnati and *Columbus Standards*

HIGH SCHOOL GEOMETRY

<p style="text-align: center;">Unit 1: Congruence, Proof, and Constructions</p>

Congruence

- G-CO-1* Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- G-CO-2* Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO-3* Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO-4* Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO-5* Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
- G-CO-6* Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- G-CO-7* Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- G-CO-8* Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
- G-CO-9* Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- G-CO-10* Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- G-CO-11* Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

- G-CO-12* Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Creating congruent segments; creating congruent angles; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- G-CO-13* Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 2:
Similarity, Proof, and Trigonometry

Similarity, Right Triangles, and Trigonometry

- G-SRT.1* Verify experimentally the properties of dilations given by a center and a scale factor:
- G-SRT.1a* A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- G-SRT.1b* The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- G-SRT.2* Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- G-SRT.3* Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
- G-SRT.4* Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
- G-SRT.5* Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- G-SRT.6* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- G-SRT.7* Explain and use the relationship between the sine and cosine of complementary angles.
- G-SRT.8* Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Modeling with Geometry

- G.MG.1* Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- G.MG.2* Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
- G.MG.3* Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Similarity, Right Triangles, and Trigonometry

- G-SRT.9* (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- G-SRT.10* (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- G-SRT.11* (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 3:
Extending to Three Dimensions

Geometric Measurement and Dimension

- G-GMD.1* Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
- G-GMD.3* Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- G-GMD.4* Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry

- G.MG.1* Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Unit 4:
Connecting Algebra and Geometry Through Coordinates

Expressing Geometric Properties with Equations

- G-GPE.2* Derive the equation of a parabola given a focus and directrix.
- G-GPE.4* Use coordinates to prove simple geometric theorems algebraically.
- Example: prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.
- G-GPE.5* Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- G-GPE.6* Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- G-GPE.7* Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

<p style="text-align: center;">Unit 5: Circles With and Without Coordinates</p>

Circles

- G-C.1* Prove that all circles are similar.
- G-C.2* Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- G-C.3* Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- G-C.4* (+) Construct a tangent line from a point outside a given circle to the circle.
- G-C.5* Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations

- G-GPE.1* Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- G-GPE.4* Use coordinates to prove simple geometric theorems algebraically.
- Example: prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

Modeling with Geometry

- G.MG.1* Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Unit 6: Applications of Probability
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Conditional Probability and the Rules of Probability

- S-CP.1* Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- S-CP.2* Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP.3* Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A , and the conditional probability of B given A is the same as the probability of B .
- S-CP.4* Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
- Example: collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
- S-CP.5* Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- Example: compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
- S-CP.6* Find the conditional probability of A given B as the fraction of B 's outcomes that also belong to A , and interpret the answer in terms of the model.
- S-CP.7* Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
- S-CP.8* (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
- S-CP.9* (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions

- S-MD.6* (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.7* (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note: “All college and career ready standards (those without a +) are found in each pathway. A few (+) standards are included to increase coherence but are not necessarily expected to be addressed on high stakes assessments.”

(http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf, p.2)

The (+) standards are expected to be taught in an honors or accelerated Geometry course.

*From: NCTM, *CCSS, *KDE, Archdioceses of *Cincinnati and *Columbus Standards*

HIGH SCHOOL ALGEBRA II

Unit 1:

Polynomial, Rational, and Radical Relationships

The Complex Number System

- N-CN.1* Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.2.* Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- N-CN.7* Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.9* (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Seeing Structure in Expressions

- A-SSE.1* Interpret expressions that represent a quantity in terms of its context.
- A-SSE.1a* Interpret parts of an expression, such as terms, factors, and coefficients.
- A-SSE.1b* Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Example: interpret $P(1+r)^n$ as the product of P and a factor not depending on P .
- A-SSE.2* Use the structure of an expression to identify ways to rewrite it.
- Example: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Arithmetic with Polynomials and Rational Expressions

- A-APR.1* Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-APR.3* Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.7*. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities

- A-REI.2* Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- A-REI.11* Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, and absolute value functions.

Interpreting Functions

- F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-IF.7c* Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

<p style="text-align: center;">Unit 2: Vector and Matrix Quantities</p>

Perform operations on matrices and use matrices in applications

- N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.
- N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vector.

Unit 3: Modeling with Functions
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Creating Equations

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational functions.*
- A-CED.2* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3* Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- Example: represent inequalities describing nutritional and cost constraints on combinations of different foods.
- A-CED.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
- Example: rearrange Ohm's law $V = IR$ to highlight resistance R .

Interpreting Functions

- F-IF.4* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- F-IF.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- Example: if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
- F-IF.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.7* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-IF.7b* Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- F-IF.8* Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-IF.9* Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Example: given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

- F.BF.1* Write a function that describes a relationship between two quantities.
- F.BF.1b* Combine standard function types using arithmetic operations.
- Example: build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F.BF.3* Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4* Find inverse functions.

Unit 4:
Inferences and Conclusions from Data

Interpreting Categorical and Quantitative Data

- S-ID.4* Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions

- S-IC.1* Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-IC.2* Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
- Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

- S-IC.3* Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- S-IC.4* Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S-IC.5* Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S-IC.6* Evaluate reports based on data.

Using Probability to Make Decisions

- S-MD.6* (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- S-MD.7* (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note: The (+) standards are expected to be taught in an honors or accelerated course.

*From: NCTM, *CCSS, *KDE, Archdioceses of *Cincinnati and *Columbus Standards*

HIGH SCHOOL PRE-CALCULUS

Number and Quantity

The Complex Number System

Perform arithmetic operations with complex numbers.

N-CN.3* (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

N-CN.4* (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5* (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers. Geometrically on the complex plane; use properties of this representation for computation.

□ Example: $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120° .

N-CN.6* (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equal.

N-CN.8* (+) Extend polynomial identities to the complex numbers.

□ Example: rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

N-CN.9* (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

Represent and model with vector quantities.

N-VM.1* (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v , $|v|$, $\|v\|$, v).

- N-VM.2* (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- N-VM.3* (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

- N-VM.4* (+) Add and subtract vectors.
- N-VM.4a* (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- N-VM.4b* (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- N-VM.4c* (+) Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w , with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- N-VM.5* (+) Multiply a vector by a scalar.
- N-VM.5a* (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
- N-VM.5b* (+) Compute the magnitude of a scalar multiple cv using $\|cv\| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

- N-VM.12* (+) Work with 2×2 matrices s transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Seeing Structure in Expressions

- A-SSE.4* Derive the formula for the sum of a finite arithmetic series, arithmetic sequences, and geometric sequence (when the common ratio is not 1), and use the formula to solve problems.
- Example: calculate mortgage payments.

Arithmetic with Polynomials and Rational Expressions

Understand the relationship between zeros and factors of polynomials.

A-APR.2* Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Use polynomial identities to solve problems.

A-APR.4* Prove polynomial identities and use them to describe numerical relationships.

- Example: the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A-APR.5* (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

A-APR.6* Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system

Reasoning with Equations and Inequalities

Solve systems of equations.

A-REI.8* (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Functions

Interpreting Functions

Analyze functions using different representations.

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-IF.7e* Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

F-BF.1a (+) Compose functions.

- Example: If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

- F-BF.4* Find inverse functions.
- F.BF.4a* Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
- Example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.
- F-BF.4b* (+) Verify by composition that one function is the inverse of another.
- F-BF.4c* (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- F-BF.4d* (+) Produce an invertible function from a non-invertible function by restricting the domain.
- F-BF.5* (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Create Equations.

- A-CED.1* Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational functions.*

Linear, Quadratic, and Exponential Models

- F-LE.4* For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

- F-TF-1* Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

- F-TF.2* Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.3* (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
- F-TF.4* (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

- F-TF.5* Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F-TF.6* (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- F-TF.7* (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

- F-TF.8* Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- F-TF.9* (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Geometry

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

- G-SRT.9* (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- G-SRT.10* (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- G-SRT.11* (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles

Understand and apply theorems about circles.

G-C.4* (+) Construct a tangent line from a point outside a given circle to the circle.

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section

G-GPE.3* (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

G-GPE.3a (+) Use equations and graphs of conic sections to model real-world problems

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

G-GMD.2* (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD.4* Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Note: (+) indicates standard beyond College and Career Ready.

*From: CCSS, *Massachusetts Curriculum Framework for Mathematics, March 2011*

AP Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central [apcentral.collegeboard.org] for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs.

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process.
- Calculating limits using algebra.
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior. • Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits.
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically.
- Derivative interpreted as an instantaneous rate of change.
- Derivative defined as the limit of the difference quotient.
- Relationship between differentiability and continuity.

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function

- Corresponding characteristics of graphs of f and f' .
- Relationship between the increasing and decreasing behavior of f and the sign of f' .
- The Mean Value Theorem and its geometric interpretation.
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f , f' , and f'' .
- Relationship between the concavity of f and the sign of f'' .
- Points of inflection as places where concavity changes.

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity.
- Optimization, both absolute (global) and relative (local) extrema.
- Modeling rates of change, including related rates problems.
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- Derivative rules for sums, products, and quotients of functions.
- Chain rule and implicit differentiation.

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums.
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity).

Applications of integrals.

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions.
- Antiderivatives by substitution of variables (including change of limits for definite integrals).

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Numerical approximations to definite integrals.

Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

From: The College Board, 2012, www.collegeboard.org.

For AP Calculus AB resources including sample syllabi and modules:

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

Appendix A

LINKS TO INTERNET RESOURCES

4Kids: www.4kids.org

A Plus Math: www.aplusmath.com

AP Calculus AB resources including sample syllabi and modules:

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

Archdiocese of Cincinnati Catholic School's Pre-K Math Standards:

<http://www.catholiccincinnati.org/wp-content/uploads/2012/02/MathPre-K.pdf>

Awesome Library Teacher Resources: www.awesomelibrary.org

Better Lesson: <http://betterlesson.com/home>

Common-Core Math In Practice: www.edweek.org/ew/collections/common-core-math-report2014/index.html?cmp=enl-eu-news1

Common Core Math Resources:

<http://www.commoncoreconversation.com/mathresources.html#sthash.5F06eICi.QVE2pq0J.dpbs>

Common Core Standards: www.corestandards.org

Common Core Video Exemplars: http://www.teachingthecore.org/common_core_videosCool

Daily Calendar Activities: www.greatsource.com/everydaycounts

Digital Textbooks (free): www.ck12.org/?gclid=ciw8x_gp-mmcf7m7aodmfmajq

Education Place: www.eduplace.com

Eisenhower National Clearinghouse for Mathematics and Science Education (ENC):

www.enc.org

eTextbooks: www.hmheducation.com/etextbooks/

Fact Monster: www.factmonster.com

Go Math Program: www.hmheducation.com/gomath

Great Sites for Kids: www.greatsitesforkids.com

How To Smile Math Activities: www.howtosmile.org

Illustrative Mathematics: www.illustrativemathematics.org

K-12 Mathematics Curriculum: www.edc.org/mcc

Khan Academy LEARNSTORM: www.khanacademy.org/learnstorm

MICCSS - Michigan CCSS Resources for Teachers: www.miccss.org

Math: www.coolmath4kids.com

Math Expressions Program: www.hmhededucation.com/mathexpressions/

Math Kid Sites: www.kidsites.com

Math Tool Box – curriculum frameworks: <http://www.ccsstoolbox.org/?3e3ea140>

NCTM: www.nctm.org

National Council of Teachers of Mathematics:
www.nctm.org/flipbooks/standards/pssm/index.html

National Governors Association for Best Practices: www.nga.org/cms/center/edu

Ohio Resource Center for Mathematics, Science and Reading: www.ohiorc.org/for/math/

On Core Mathematics: www.hmhededucation.com/mathexpressions/

PARCC “Take the Test”: www.parcconline.org/take-the-test

Pearson Math Practice Tests: www.parcc.pearson.com/practice-tests/math

Professional Development: www.hmhededucation.com/professionaldevelopment/specialized_training_math.php

Professional Development: www.leadandlearn.com

Secret Millionaires Club: www.smckids.com

Soar to Success Math Intervention: www.hmhededucation.com/mathexpressions/

Student Engage NY Math Books:
www.fwps.org/teaching/studentonlinelearning/studentengageny-math-books/

Summer Success Math: www.hmhededucation.com/summerschool/summer_success_math.php

Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students:
http://ies.ed.gov/ncee/wwc/pdf/practice_guides/wwc_algebra_040715.pdf

Tools for Math Educators: www.insidemathematics.org

Unit Development Guidelines: <http://cici-online.org/cici-design/unit-development/unitdevelopment-guidelines>

Yay Math!: www.yaymath.org/video.html

Resources for Parents

Alabama State Department of Education Standards:

<http://www.alsde.edu/sec/comm/pages/standardoftheweek-all.aspx>

Great Schools Math Parent Resources:

<http://www.greatschools.org/gk/category/academics/math-2/>

Math Milestones: www.greatschools.org/gk/milestones/

PTA Parent's Guide to Student Success:

<http://www.pta.org/parents/content.cfm?ItemNumber=2583&navItemNumber=3363>

Parent Roadmaps to the Common Core Standards- Mathematics:

www.cgcs.org/page/244?utm_source=parcc+updates+11%2f13%2f14&utm_campaign=11%2f13%2f2014+update&utm_medium=email

Appendix B

ASSESSMENT

Evaluation is fundamental to the process of making this curriculum a reality.

This curriculum proposes that --

1. student assessment be integral to instruction
 2. multiple means of assessment methods be used
 3. all aspects of mathematical knowledge and its connection be assessed
 4. instruction and curriculum be considered equally in judging the quality of a program
-

GENERAL ASSESSMENT

The vision of mathematics education places new demands on instruction and forces us to reassess the manner and methods by which we chart our students' progress. In an instructional environment that demands a deeper understanding of mathematics, testing instruments that call for only the identification of single correct responses no longer suffice. Instead, our instruments must reflect the scope and intent of our instructional program to have students solve problems, reason, and communicate. Furthermore, the instruments must enable the teacher to understand students' perceptions of mathematical ideas and processes and their ability to function in a mathematical context. At the same time, they must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction.

Many assessment techniques are available:

1. portfolios
2. multiple-choice
3. short-answer
4. discussion or open-ended questions
5. structured or open-ended interviews
6. homework
7. projects
8. journals
9. essays
10. dramatizations and class presentations

Among these techniques are those appropriate for students working in whole-class settings, in small groups, or individually. The mode of assessment can be written, oral, or computer oriented. Assessment should not rely on a single instrument or technique.

Appendix C

MANIPULATIVES

The heart of an effective, well-taught mathematics program is the careful development of concepts and skills through extensive use of concrete material. Students must be actively involved in constructing, modifying and integrating ideas by interacting with physical material. They must investigate quantitative and spatial situations by manipulating the materials and then translating the ideas through pictures or diagrams to corresponding abstract symbolic representations. These are essential elements of a program in which mathematics is done with understanding rather than rote.

It is not sufficient to confine the use of concrete materials to teacher demonstration; each student must manipulate his/her own materials. K-8 classrooms especially must be equipped with a wide variety of physical materials and supplies. Manipulatives should continue to be used in grades 9-12 whenever appropriate.

Classrooms should have quantities of materials such as:

- counters
- interlocking cubes
- connecting links
- bean sticks and/or bundling sticks
- base ten, attribute, and pattern blocks
- geometric models
- rulers and other measuring devices
- spinners
- color rods
- geoboards
- balance scales
- fraction pieces
- graph, grid and dot paper
- upper grades should also have compasses and protractors
- geometric puzzles such as tangrams and pentaminoes

Appendix D

MATH LITERATURE SUGGESTIONS

Kindergarten:

- *What Comes in 2's, 3's, and 4's?* by Suzanne Aker
- *Pattern Bugs* by Trudy Harris
- *Swan Harbor: A Nature Counting Book* by Laura Rankin
- *All for Pie, Pie for All* by David Martin
- *One Stuck Duck* by Phyllis Root
- *The Pig Is in the Pantry, The Cat Is on the Shelf* by Shirley Mozelle
- *Anno's Counting Book* by Mitsumasa Anno (ISBN 0064433150)
- *The Three Little Pigs* by Paul Galdone (ISBN 0064431231)
- *The Very Hungry Caterpillar* by Eric Carle (ISBN 0399208539)
- *Ten Red Apples* by Pat Hutchins (ISBN 0688167974)
- *Afro-Bets* by Cheryl Willis (ISBN 0940975019)
- *The Very Busy Spider* by Eric Carle (ISBN 059412902)
- *My First Look at Numbers* by Toni Ran (ISBN 067905335)
- *And the Cow Said Moo* by Mildred Phillips (ISBN 0688168032)
- *Bear in a Square* by Stella Blackstone (ISBN 1846860553)
- *Shapes* by Margie Burton (ISBN 1892393379)
- *The Shape of Things* by Dayle Ann Dodds (ISBN 0439136662)
- *Afro-Bets* by Cheryl Willis (ISBN 0940975019)
- *Math Counts* by Henry Pluckrose (ISBN 0516454566)
- *Number Munch! A Funny Crunchy Counting Book* by Chuck Reasoner (ISBN 0843136746)
- *Time To...* by Bruce McMillan (ISBN 0688088552)

Grade 1:

- *Dinner at the Panda Palace* by Stephanie Calmenson
- *Rumble Bus* by Larry Dane Brimmer
- *How Many Birds?* by Don L. Curry
- *100 Days of School* by Trudy Harris
- *Tail Feather Fun: Counting by Tens* by Michael Dahl
- *The Great Graph Contest* by Loreen Leedy
- *How Do You Know What Time It Is?* by Robert E. Wells
- *Caps for Sale* by Esphyr Slobodkina
- *More Bugs? Less Bugs?* by Don L. Curry
- *When a Line Bends . . . A Shape Begins* by Rhonda Gowler Greene
- *Dos, Tres: One, Two, Three* by Pat Mora (ISBN 0618054685)

- *A Triangle for Adaora* by Ifeoma Onyefulu (ISBN 1845077385)
- *Round is a Mooncake* by Roseanne Thong (ISBN 0811826767)
- *Circles and Squares Everywhere!* by Max Grover (ISBN 0152000917)
- *So Many Circles, So Many Squares* by Tana Hoban (ISBN 0688151655)
- *Unos, Dos, Tres: One, Two, Three* by Pat Mora (ISBN 0618054685)
- *A Triangle for Adaora* by Ifeoma Onyefulu (ISBN 1845077385)
- *Round is a Mooncake* by Roseanne Thong (ISBN 0811826767)
- *Circles and Squares Everywhere!* by Max Grover (ISBN 0152000917)
- *Hopscotch Around the World* by Mary Lankford (ISBN 0688147453)
- *Emeka's Gift: An African Counting Story* by Ifeoma Onyefulu (ISBN 0711214476)
- *Feast for Ten* by Cathryn Falwell (ISBN 0547064314)
- *I Knew Two Who Said Moo* by Judi Barrett (ISBN 068985935X)
- *One Little Mouse* by Dori Chaconas (ISBN 0670889474)
- *Counting Our Way to Maine* by Maggie Smith (ISBN 0892727756)
- *One Moose, Twenty Mice* by Clare Beaton (ISBN 1841481297)
- *Count on Your Fingers African Style* by Claudia Zaslavsky (ISBN 0863162509)
- *I Spy Two Eyes* by Lucy Micklethwait (ISBN 0688126405)
- *Turtle Splash! Countdown at the Pond* by Cathryn Falwell (ISBN 0061429279)
- *Ten Flashing Fireflies* by Philemon Sturges (ISBN 1558586741)
- *Elevator Magic* by Stuart Murphy (ISBN 0064467090)
- *Animals on Board* by Stuart Murphy (ISBN 0064467163)
- *Too Many Dinosaurs* by Bob Barner (ISBN 0553375660)
- *Math Fables: Lessons That Count* by Greg Tang (ISBN 0439754984)
- *Domino Addition* by Lynette Long (ISBN 0881063525)
- *Adding It Up at the Zoo* by Judy Nayer (ISBN 0736812784)
- *Subtraction Fun* by Betsy Franco (ISBN 0736812873)
- *Spunky Monkeys on Parade* by Stuart Murphy (ISBN 0064467279)
- *Two Ways to Count to Ten* by Ruby Dee (ISBN 0805013148)
- *Underwater Counting: Even Numbers* by Jerry Pallotta (ISBN 0881068004)
- *One Guinea Pig is Not Enough* by Kate Duke (ISBN 014056814X)
- *Mission: Addition* by Loreen Leedy (ISBN 0823414124)
- *A Chair for My Mother* by Vera Williams (ISBN 0688040748)
- *My Rows and Piles of Coins* by Tololwa Mollel (ISBN 0395751861)
- *Picking Peas for a Penny* by Angela Medearis (ISBN 0938349554)
- *Mama Bear* by Chyng Feng Sun (ISBN 0395634121)
- *One Hundred Is a Family* by Pam Ryan (ISBN 078681120X)
- *Chicka Chicka 1, 2, 3* by Bill Martin (ISBN 0689858817)
- *Many Ways to 100* by Betsy Franco (ISBN 0736870326)
- *The Best Vacation Ever* by Stuart Murphy (ISBN 0064467066)
- *The Birth-Order Blues* by Joan Drescher (ISBN 0670836214)
- *Lemonade for Sale* by Stuart Murphy (ISBN 0064467155)

- *Graph It* by Lisa Trumbauer (ISBN 0736812822)
- *A Cloak for the Dreamer* by Aileen Friedman (ISBN 0590489879)
- *Lots and Lots of Zebra Stripes* by Stephen Swinburne (ISBN 1563979802)
- *The Seasons Sewn* by Ann Whitford Paul (ISBN 0152021078)
- *I Spy Shapes in Art* by Lucy Micklethwait (ISBN 0060731931)
- *Cubes, Cones, Cylinders, & Spheres* by Tana Hoban (ISBN 0688153259)
- *Two of Everything: A Chinese Folktale* by Lily Toy Hong (ISBN 0807581577)
- *Anno's Magic Seeds* by Mitsumasa Anno (ISBN 0698116186)
- *Kites: Magic Wishes That Fly Up to the Sky* by Demi (ISBN 0375810080)
- *Less Than Half! More Than Whole* by Kathleen Lacapa (ISBN 0873585925)
- *The Turtle and the Monkey* by Joanna Galdone (ISBN 0395544254)
- *Give Me Half!* by Stuart Murphy (ISBN 0064467015)
- *Eating Fractions* by Bruce McMillan (ISBN 0590437712)
- *Tell Me What the Time Is* by Shirley Willis (ISBN 0531159795)
- *Somewhere in the World Right Now* by Stacey Schuett (ISBN 0679885498)
- *Game Time!* by Stuart Murphy (ISBN 0064467325)
- *The Grapes of Math* by Greg Tang (ISBN 0439598400)
- *The Smushy Bus* by Leslie Helakoski (ISBN 0761319174)
- *Fifty-Five Grandmas and a Llama* by Lynn Manuel (ISBN 0879057858)
- *The Coin Counting Book* by Rozanne Lanczak (ISBN 0881063258)

Grade 2:

- *Domino Addition* by Lynette Long
- *Keep Your Distance!* by Gail Herman
- *Count on Pablo* by Barbara deRubertis
- *A Cloak for a Dreamer* by Aileen Friedman
- *A Place for Zero: A Math Adventure* by Angeline Sparanga LoPresti
- *How Do You Know What Time It Is?* by Robert E. Wells
- *Fair is Fair!* by Jennifer Dussling
- *A Cloak for the Dreamer* by Aileen Friedman
- *Henry Hikes to Fitchburg* by D.B. Johnson
- *Patterns in Nature* by Jennifer Rozines Roy and Gregory Roy
- *One Grain of Rice: A Mathematical Folktale* by Demi
- *Measuring Penny* by Loreen Leedy
- *The Grapes of Math* by Greg Tang
- *Henry Hikes to Fitchburg* by D.B. Johnson
- *Oliver's Party* by Jenny Fry (ISBN 0764154614)
- *Jelly Beans for Sale* by Bruce McMillan (ISBN 0590865968)
- *Racing Around* by Stuart Reed (ISBN 0064462440)
- *Even Steven and Odd Todd* by Kathryn Cristaldi (ISBN 0590227155)
- *Shapes (Slide 'n Seek)* by Chuck Murphy (ISBN 0689844775)

- *Shapes, Shapes, Shapes* by Tana Hoban (ISBN 0688147402)
- *Hippos Go Berserk* by Sandra Boynton (ISBN 0689808180)
- *Clocks and More Clocks* by Pat Hutchins (ISBN 0689717697)
- *My Grandmother's Clock* by Geraldine McGaughrean (ISBN 0618216956)
- *Tiger Math: Learning to Graph a Baby Tiger* by Ann Whitehead Nagda (ISBN 0805071610)
- *Tightwad Tod* by Daphne Skinner (ISBN 1575651095)
- *100th Day Worries* by Margery Cuyler (ISBN 1416907893)
- *Pigs Will Be Pigs: Fun With Math and Money* by Sharon McGinley (ISBN 0689812194)
- *Earth Day-Hooray* by Stuart Murphy (ISBN 0060001292)
- *The 329th Friend* by Marjorie Weinman (ISBN 0590315487)
- *Hannah's Collections* by Marthe Jocelyn (ISBN 0887766909)
- *Tell Me How Far It Is* by Shirley Willis (ISBN 0531159750)
- *Millions to Measure* by David Schwartz (ISBN 0060848064)
- *Each Orange Had 8 Slices: A Counting Book* by Paul Giaganti (ISBN 0688139858)
- *The Door Bell Rang* by Pat Hutchins (ISBN 0874998030)
- *Let's Fly a Kite* by Stuart Murphy (ISBN 0064467377)

Grade 3:

- *How Much, How Many, How Far, How Heavy, How Long, How Tall Is 1000?* by Helen Nolan
- *Grandfather Tang's Story: A Tale Told with Tangrams* by Ann Tompert
- *One Less Fish* by Kim Michelle Toft and Allen Sheather
- *Sam Johnson and the Blue Ribbon Quilt* by Lisa Campbell Ernst
- *Amanda Bean's Amazing Dream: A Mathematical Story* by Cindy Neuschwander
- *One Hundred Hungry Ants* by Elinor J. Pinczes
- *A Grain of Rice* by Helena Clare Pittman
- *Building with Shapes* by Rebecca Weber
- *The Doorbell Rang* by Pat Hutchins
- *Jumanji* by Chris Van Allsburg
- *Fraction Fun* by David A. Adler
- *Building with Shapes* by Rebecca Weber
- *Penguins at Home: Gentoos of Antarctica* by Bruce McMillan
- *Alice Ramsey's Grand Adventure* by Don Brown
- *A Bundle of Beast* by Mark Steele (ISBN 0395616208)
- *The Greatest Gymnast of All* by Stuart Murphy (ISBN 0007619351)
- *One Grain of Rice* by Demi (ISBN 0590939980)
- *Amanda Bean's Amazing Dream* by Cindy Neuschwander (ISBN 0590300124)
- *Spaghetti and Meatballs for All: A Mathematical Story* by Marilyn Burns (ISBN 0545044455)
- *Sea Squares* by Joy N. Hulme (ISBN 1562825201)

- *Secret Treasures and Mathematical Measures: Adventures in Measuring: Time, Temperature, Length, Weight, Volume, Angles, Shapes, and Money* by Chris Kensler (ISBN0743235259)
- *The Big Orange Splot* by Daniel Manus Pinkwater (ISBN 0590445108)
- *Jump, Kangaroo, Jump* by Stuart Murphy (ISBN 0064467216)
- *Mega-Fun Fractions* by Martin Lee (ISBN 0439288446)
- *The Fraction Family Heads West* by Marti Dryk (ISBN 0965891219)
- *Piece=Part=Portion: Fractions=Decimals=Percents* by Scott Gifford (ISBN 1582462615)
- *A Remainder of One: A Mathematical Folktale* by Elinor J. Pinczes (ISBN 0618250776)
- *The Great Divide* by Dayle Ann Dodds (ISBN 0763615925)
- *Room for Ripley* by Stuart Murphy (ISBN 0395665602)
- *How Tall, How Short, How Faraway?* (ISBN 0823413751)

Grade 4:

- *Amanda Bean's Amazing Dream: A Mathematical Story* by Cindy Neuschwander
- *Anno's Mysterious Multiplying Jar* by Masaichiro and Mitsumasa Anno
- *Grandfather Tang's Story: A Tale Told with Tangrams* by Ann Tompert
- *Sold! A Mathematics Adventure* by Nathan Zimelman
- *A Fly on the Ceiling: A Math Reader* by Julie Glass
- *Rabbits Rabbits Everywhere: A Fibonacci Tale* by Ann McCallum
- *The Math Chef* by Joan D'Amico and Karen Eich Drummond
- *A Remainder of One* by Elinor J. Pinczes
- *Tiger Math: Learning to Graph from a Baby Tiger* by Ann Whitehead Nagda and Cindy Bickel
- *Fraction Action* by Loreen Leedy
- *Building with Shapes* by Rebecca Weber
- *Pigs Will Be Pigs: Fun with Math and Money* by Amy Axelrod
- *Hottest, Coldest, Highest, Deepest* by Steve Jenkins
- *Arithme-Tickle: An Even Number of Odd Riddle-Rhymes* by Patrick Lewis (ISBN 0152058487)
- *The Kings Chessboard* by David Birch (ISBN 0140548808)
- *The Great Divide: A Mathematical Marathon* by Dayle Ann Dodds (ISBN 0763615925)
- *The Math Chef* by Joan D'Amico and Karen Eich Drummond (ISBN 0471138136)
- *Sir Cumference and the First Round Table: A Math Adventure* by Cindy Neuschwander (ISBN 1570911606)
- *Careless at the Carnival: Junior Discovers Spending* by Dave Ramsey (ISBN 0972632317)

Grade 5/6:

- *Anno's Mysterious Multiplying Jar* by Masaichiro and Mitsumasa Anno
- *O, Say Can You See?* by Sheila Keenan
- *Building with Shapes* by Rebecca Weber
- *Polar Bear Math: Learning About Fractions from Klondike and Snow* by Ann Whitehead Nagda and Cindy Bickel
- *Sir Cumference and the Great Knight of Angleland: A Math Adventure* by Cindy Neuschwander
- *Fraction Action* by Loreen Leedy
- *The Math Chef* by Joan D'Amico and Karen Eich Drummond
- *The Amazing Impossible Erie Canal* by Cheryl Harness
- *Tiger Math: Learning to Graph from a Baby Tiger* by Ann Whitehead Nagda and Cindy Bickel
- *Go, Fractions!* by Judith Bauer Stamper
- *Find the Constellations* by H.A. Rey
- *Piece = Part = Portion: Fractions = Decimals = Percents* by Scott Gifford
- *The Best of Times: Math Strategies that Multiply* by Greg Tang
- *Math-terpieces: The Art of Problem Solving* by Greg Tang (ISBN 043943883)
- *One Grain of Rice: A Mathematical Folktale* by Demi (ISBN 0590939980)
- *Sir Cumference and the Dragon of Pi* by Cindy Neuschwander (ISBN 1570911644)
- *7x9=Trouble* by Claudia Mills (ISBN 0374464523)
- *Jumanji* by Chris Van Allsburg (ISBN 0395304488)
- *Mummy Math: An Adventure in Geometry* by Cindy Neuschwander (ISBN 0805075052)

From: Archdiocese of Cincinnati's math resources

Resources

ACT College and Career Ready Math Standards: <http://www.act.org/standard/planact/math/>

Archdiocese of Cincinnati Graded Course of Study for Mathematics, 2012:
<http://www.catholiccincinnati.org/wp-content/uploads/2012/02/MathCurriculumComplete.pdf>

Archdiocese of Cincinnati Graded Course of Study for High School Mathematics, Algebra I, Geometry, 2014: <http://www.catholiccincinnati.org/wp-content/uploads/2014/05/GEOMETRYCOMPLETE-FILE.pdf>

Archdiocese of Louisville Mathematics Curriculum Framework, 2011:
<http://www.archlou.org/wp-content/uploads/2011/06/Mathematics-Curriculum-Framework.pdf>

Archdiocese of Milwaukee Curriculum Guides:
<http://schools.archmil.org/schools/about/curriculum.htm>

AP Calculus AB Course Description:
<http://media.collegeboard.com/digitalServices/pdf/ap/apcalculus-course-description.pdf>

Common Core Math Standards: www.corestandards.org/Math/

CCSS Mathematics Appendix A:
http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf

Diocese of Columbus Mathematics Course of Study 2012:
<http://www.cdeducation.org/Portals/3/OCS/docs/COS/Mathematics%20Course%20of%20Study%20-%202012.pdf>

Iowa Tests of Basic Skills Content Classifications, 2008, Riverside Publishing Company

KDE Core Standards for Math:
<http://education.ky.gov/curriculum/standards/Documents/Kentucky%20Common%20Core%20MATHMATICS.pdf>

National Council of Teachers of Mathematics (2014), *Principles to Actions*. Reston, VA: NCTM, Inc., <http://nctm.org>

Engage^{ny} CCSS Library: <https://www.engageny.org/ccss-library>

Massachusetts Curriculum Framework for Mathematics, PK-12:
<http://www.doe.mass.edu/frameworks/math/0311.pdf>